IJREAT International Journal of Research in Engineering & Advanced Technology, Volume 7, Issue 5, Oct - Nov, 2019
ISSN: 2320 – 8791 (Impact Factor: 2.317)
www.ijreat.org

STATISTICAL APPLICATIONS OF CONFOUNDING TECHNIQUES IN FACTORIAL DESIGNS FOR BASIC SCIENCE AND ENGINEERING

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Abstract

In Statistical Confounding models Research Analysis we can able to understand the concept, importance and technique of confound of factorial experiments. Evaluation of 2^2, 2^3 , 3^2Frequency Distributions are more authentic in test Statistics. Determination of different confounding models through Testing of Hypothesis, Partial confounding, Total Confounding,Balanced partial confound will provided the importance of factorial designs in Basic Science and Engineering Curriculum. We can understand the confounding effects of factorial experiment. Partially obtained partial information regarding the effects confounded from the replicates in which the effects are confounded. Balanced partial confounding of an effect will influence the Test Statics in Confounding Designs. In Statistical Analysis of Confound ANOVA provides the technique to simplify the computation for Design of experiments. One way Classification, Design of Experiments, and Testing of Hypothesis where it involves confounding effective ness and Treatments are plays a vital role in its application process.

Key words: confounding, determination, factorial experiment, hypothesis, partial confounding, replicates.

I. INTRODUCTION

Confounding is a method for forming factorial experiment in blocks. The number of treatment combination increases rapidly as the number of factors or as the number of levels of each factor is increased. In a Design, if different effects are confounded in different replicates the design is known as partially confounded design.

Partial information may be desired on all effects in a factorial experiment with a large number of treatments, and at the same time a small block size may be desirable at the particular stage. A certain approach is followed where some effects are confounded one of the replicate, similarly second replicate and continues for further remaining replicates. The Treatments means are always adjusted in partially confounded arrangements. The reason for adjustment is that estimates of effects are obtainable and hence the effect of the incomplete block is estimable, resulting in a more reliable estimate of the total treatments. The technique consists in splitting up of each replicate into a number of incomplete blocks containing an equal number of plots and allocating the treatment combinations to these block in a way that ensures the orthogonality of the contrast due to these block effects with the unconfounded treatment effects.

II. BASIC APPROACHES IN CONFOUNDING

It is Design of approach for arranging a factorial experiment in blocks. It consists small number of treatment combinations in one replicate. In general we can consider the two types of confounded models.

i) Partially confounded Design

ii) Totally confounded Design

If an effect is zero interest or very small interest this effect may be confounded with the incomplete block differences in all the replicates. This system of confounding is termed as total confounding (T.C.) If one effect is confounded with incomplete block differences in one or more replicates, another effect is confounded in one or more replicates and so on, then these effects are partially confounded with incomplete block defenses. In this Process some information is available on all treatment comparisons though some comparisons are more accurately determined, since the information on these are available from all the replicates. This system of confounding is termed as Partial confounding (P.C). ifthere exists 2 factors with some differences in same number of replicates in a certain model then the model is termed as Partially confounded design.

III. ADVANTAGES AND DISADVANTAGES OF CONFOUNDING

- The major advantage of confound lies in the fact that it reduces the experimental errors. It divides the experimental material into homogeneous incomplete blocks.
- The removal of variation among incomplete blocks with in replicatesoften result in smaller error mean square as compared with a Complete Block Design.
- The algebraic calculations are usually difficult and the statistical analysis is complex, specially, when some of the observations are missing.
- Complications arise if treatments and blocks interact.

IV. 2^2 FACTORIAL DESIGN IN TWO BLOCKS

Considering two factors A, B each at two levels with 2^2 treatments like (1), a, b, ab. In this approach the arrangement of 4 treatment combinations 2 blocks. Block 1 contains(1) and ab and Block 2 containsa and b. In this process we can observe that the treatment combinations are in the same block randomly determined. We can randomly the which block to run initially.

METHOD:1

Bocks	1	2
	(1)	a
	ab	b

2^2 Factorial Design in two blocks

We can compute A,B without block as below:

(A') = [ab + a – b - (1)]

(B') = [ab + b– (1) – a]

In above conditions we can state that both A,B are unaffected blocking, since in each estimate there is one plus and one minus treatment combination from each block. If there exists any mutual difference then block 1 and 2 will be canceled.

If we consider (AB) ' = [ab + (1) – a - b]

AB Is confounded with blocks. Hence from this it is apparent that one can use the table of plus and minus signs for the 2^2 design.

Treatment's	I	A	B	AB
(1)	+	-	-	+
a	+	+	-	-
b	+	-	+	-
ab	+	+	+	+

Table of '+' and '-'

- The treatment combinations that have Plus sign on AB are assigned to block-1, (1) and b had been assigned to block-1.
- The treatment combinations which have Minus sign on AB are assigned to block-2, a and ab had been assigned to block-2.
- This entire the process of design is used to confound any 2^k design in two-blocks.

METHOD:2

Considering confounded Effect AB, then corresponding Equations for the blocks are

$$L = \lambda_1 x_1 + \lambda_2 x_2 \ (\text{mod } 2)$$

$$L = x_1 + x_2 (\text{mod}2)$$

Treatment Combinations			$L = x_1 + x_2(\text{mod}2)$
	A	B	
(1)	0	0	0+0=0
a	1	0	1+0=1
b	0	1	0+1=1
ab	1	1	1+1=2=0

If we consider 4 replicates of 2^2 F.D. with 2 blocks each with effect AB confounded in all the replicates.

Repllicate-1 : Block-1 consists (1) & ab
Block-2 consistsa& b

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Repllicate-2 : Block-3 consists (1) & ab, where L=0
Block-4 consistsa & b, where L=1

Repllicate-3 : Block-5 consists (1) & ab, where L=0
Block6 consistsa & b, where L=1

Repllicate-4 : Block-7 consists (1) & ab, where L=0
Block-8 consistsa & b , where L=1

V. TESTING OF HYPOTHESES

In Testing of Hypotheses we consider Null Hypothesis as Confound is not effect and treatments are homogenous. Using ANOVA modules we apply the following steps to verify the Hypotheses condition.

- Grand total $G = \sum\sum\sum Y_{ijk}$ for alli,j,k values.
- Correction factor = G^2 / N ; $N = 4*2^2 = 16$.
- Block $S^2 = S_B^2 = 1/2[B_1^2 + \ldots B_8^2] - C.F.$
  = (8 – 1) = 7 deg(f)
- Total S^2T = N-1 = 15 deg(f)
- treatments : $St^2 = SS(A) + SS(B)$ = 2 deg(f)
- Error (SSS_E^2) = $S^2T - S^2B - S^2t$
  = 15 – 7 - 2
  = 6 deg(f)

VI. 2^3 FACTORIAL DESIGN TWO BLOCKS

<u>MEHTOD-1</u> :

It consists $\mathbf{2^3}$ = 8 treatment combinations

In Block-1

(1)	ab	ac	bc

In Block-2

a	b	c	abc

Treatments	I	AB	AC	BC	ABC	a	b	c
(1)	1	1	1	1	0	0	0	0
ab	1	1	0	0	0	1	1	0
ac	1	0	1	0	0	1	0	1
bc	1	0	0	1	0	0	1	1

Block-1 Combinations

Treatments	A	B	C	ABC	I	AB	AC	BC
a	1	0	0	1	1	0	0	1
b	0	1	0	1	1	0	1	0
c	1	0	1	1	0	1	0	0
abc				1				

Block -2 Combinations

<u>METHOD-2</u>:

In Block-1 where L= 0

(1)	ab	ac	bc

In Block-2 where L=1

a	b	c	abc

Computed from the following table :

$L = k_1x_1 + k_2x_2 + k_3x_3 \pmod 2$

$L = x_3 \pmod 2 + x_2 + x_1$

Where $k_1 = k_2 = k_3 = 1$

Treatment combinations				L=X1+X2+X3(MOD2)
	A	B	C	
(1)	0	0	0	L = 0
a	1	0	0	L = 1
b	0	1	0	L = 1
ab	1	1	0	L = 2 = 0
c	0	0	1	L = 1
ac	1	0	1	L = 2 = 0
bc	0	1	1	L = 2 = 0
abc	1	1	1	L = 3 = 1

In mathematical representation we can compute the outcomes as below:

- **$ab.ac = a^2.bc = bc$; ($a^2 = 1$ as mod.2)**
- **$ab.bc = a.b^2c = ac$; ($b^2 = 1$ as mod.2**
- **$ac.bc – ab.c^2 = ab$; ($c^2 = 1$ as mod.2)**
- **b.(1) = b**
- **$b.bc. = b^2c = c$**
- **b.ac = abc**
- **$b.ab = ab^2 = a$**

VII. 2^3 TOTAL CONFOUNDING FACTORIAL DESIGN

Considering Five Replicates with ABC tabular form:

Replicate 1	Block-1	(1)	ab	ac	bc	L=0
	Block-2	a	b	c	abc	L=1
Replicate 2	Block-3	(1)	ab	ac	bc	L=0
	Block-4	a	b	c	abc	L=1
Replicate 3	Block-5	(1)	ab	ac	bc	L=0
	Block-6	a	b	c	abc	L=1
Replicate 4	Block-7	(1)	ab	ac	bc	L=0
	Block-8	a	b	c	abc	L=1
Replicate 5	Block-9	(1)	ab	ac	bc	L=0
	Block-10	a	b	c	abc	L=1

Null-hypotheses (H_0) Confounding is not effective and Treatments are homogenous.

- Grand total G = $\sum\sum\sum\sum Y_{ijkl}$for all i,j,k,l values.
- Correction factor = G^2 / N ; N= $5*2^3$ = 40.
- Block S^2 = $S_B^2 = 1/2[B_1^2 + \ldots .B_{10}^2]$ – C.F.
  = (10 – 1) = 9 deg(f)
- Total S^2T = N-1 = (40-1) -= 39 deg(f)
- Treatments : $St^2= S^2A + S^2B+ S^2C+S^2AB+S^2BC + S^2AC$; (98-1-1) = 6 deg(f)
- Error (SSS_E^2) = $S^2T - S^2_B + S^2t$
  = 39- 9- 6 =24 deg(f)

VIII. 3^2 TOTAL CONFOUNDING FACTORIAL DESIGN

In $3^{2\ \text{C.F.D}}$ we can consider 2-replicates, where in such designs we use the component of AB^2 w.r.t. to AB blocks.

$L = k_2x_2$ (mod 3)$_+$ k_1x_1

$L = 2x_2$ (mod 3)+ x_1

For $k_{1,2}$= (1,2) respectively

A	B	L = $2x_2$ (mod 3) + x_1
0	0	0
1	0	1
2	0	2
0	1	2
1	1	0
2	1	1
0	2	1
1	2	2
2	2	0

Treatments 3^2 Confounding Factorial Design

Replicate -1	Replicate - 2

L = 0 → 1st Block	L = 0 → 4th Block
L = 1 → 2nd Block	L = 1 → 5th Block
L = 2 → 3rd Block	L = 2 → 6th Block

Totally confounded 3^2 Factorial Design

	Block-1	Block-2	Block-3
Replicate -1	L = 0	L=1	
	00	10	20
	11	21	01
	22	02	12
			L = 2
Replicate -2	Block-4	Block-5	Block-6
	00	10	20
	11	21	01
	22	02	12
			L = 3

AB^2 component confounded table

A	B	$L = x_1 + x_2 \pmod 3$
0	0	L = 0
1	0	L = 1
2	0	L = 2
0	1	L = 1
1	1	L = 2
2	1	L = 3= 0
0	2	L = 2= 2
1	2	L = 3 = 0
2	2	L = 4= 1

Confounding F.D Equation Computation Table

Testing of Hypothesis consists the Null hypothesis as H_0 as confounding is not effective and Confounding Treatments are homogenous as two conditions.

IX. PARTIAL CONFOUDING

If different effects re confounded in different replicates in a design then it is know as Partial Confounded design. The treatments are always adjusted in partially confounded arrangements.

Example:

Rep I		Rep II	
BLOCK 1	BLOCK 2	BLOCK 3	BLOCK 4
(1)	a	(1)	a
ab	b	b	c
ac	c	ac	ab
bc	abc	abc	bc
ABC		AC	

The interaction ABC could be confounded with blocks in replicate-I, AC in replicate – II, BC in replicate – III and AB in replicate – IV.
The interactions are evaluated from twenty four points instead of the entire thirty two plots. The amount of information on the interactions ignoring inter

Rep III		Rep IV	
BLOCK 5	BLOCK 6	BLOCK 7	BLOCK 8
(1)	b	(1)	a
a	c	c	b
bc	ab	ab	ac
abc	ac	abc	bc
BC		AB	

block comparisons, in then 3 the relative information for the confounded effects.

X. PARTIAL CONFOUDING 2^3 F.D

Constructing 4 replicates of **2^3**F.D. in 2 blocks each with ABC, AB,AC,BC computation can be evaluated as below :

- ABC → $L_1 = x_1 + x_2 + x_3 (\text{mod } 2)$
- AB → $L_2 = x_1 + x_2 (\text{mod } 2)$

Treatments		$L_1 = x_1 + x_2 + x_3 (\text{mod } 2)$	$L_2 = x_1 + x_2 (\text{mod } 2)$
(1)	0 0 0	L1 = 0	$L_2 = 0$
a	1 0 0	L1 = 1	$L_2 = 1$
b	0 1 0	L1 = 1	$L_2 = 1$
ab	1 1 0	L1 = 0	$L_2 = 0$
c	0 0 1	L1 = 1	$L_2 = 0$
ac	1 0 1	L1 = 0	$L_2 = 1$
bc	0 1 1	L1 = 0	$L_2 = 1$
abc	1 1 1	L1 = 1	$L_2 = 0$

Similarly **:**

- AC → $L_3 = x_1 + x_3 (\text{mod } 2)$
- BC → $L_4 = x_2 + x_3 (\text{mod } 2)$

Treatments		$L_3 = x_1 + x_3 (\text{mod } 2)$	$L_4 = x_2 + x_3 (\text{mod } 2)$
(1)	0 0 0	0	0
a	1 0 0	1	0
b	0 1 0	0	1
ab	1 1 0	1	1
c	0 0 1	1	1
ac	1 0 1	0	1
bc	0 1 1	1	0
abc	1 1 1	0	0

From the above 4- Computations we can construct the layout of the design for ABC, AB,AC, and BC.

Replicates →	I		II	
Blocks	1	2	3	4
	(1)	a	(1)	a
	ab	b	ab	b
	ac	c	c	ac
	bc	abc	abc	bc
Effective Confounded	ABC		AB	

Replicates→	III		IV	
Blocks	5	6	7	8
	b	a	(1)	b
	ac	c	a	c
	(1)	ab	bc	ab
	abc	bc	abc	ac
Effective Confounded	AC		BC	

It is observed from the above table different effects are confounded different replicates in partially confounded 2^3 F.D. All the effective confounded are said to be Partially confounded.

- ABC is unconfounded in replicates II,III, and IV
- AB is unconfounded in replicates I, III and IV
- BC is unconfounded in replicates I,II and III
- AC is unconfounded in replicates I, II, and IV

Hence we get the (3/4) data about the confounded effects.

XI. TESTING OF HYPOTHESES

In Testing of Hypotheses we consider Null Hypothesis as Confound is not effect and treatments are homogenous. Using ANOVA modules we apply the following steps to verify the Hypotheses condition.

- Grand total G
- Correction factor = G^2 / N ; N= $4*2^3$ = 32.
- Block S^2 = S^2_B= $1/2[B_1^2 + \dots B_8^2]$ – C.F.
  = (8 – 1) = 7 deg(f)
- Total S^2T = N-1 = (32-1) = 31 deg(f)
- treatments : S^2t= [$S^2A + S^2B + S^2C + S^2AB + S^2AC + S^2BC + S^2ABC$]
- Error (SSS_E^2) = S^2T - S^2B - S^2t
  = 31 – 7 -7

  = 17 deg (f)

XII. Partial Confounded 3^2 F.D.

- AB^2 → $L_1 = x_1 + 2x_2$ (mod 3)
- AB → $L_2 = x_1 + x_2$ (mod3)

Treatment	0 0	1 0	2 0	0 1	1 1	2 1	0 2	1 2	2 2
	0	0	0	1	1	1	2	2	2
L1	0	1	2	2	0	1	1	0	0
L2	0	1	2	1	2	0	2		1

Computation Table for AB^2 and AB

	Replicate –I			Replicate - II		
Blocks	1	2	3	4	5	6
	00	10	20	00	10	20
	11	21	01	21	01	11
	22	02	12	12	22	02
Confounded effects	AB^2 Component			AB component		

Layout of the design for partially confounded 3^2 F.D

XIII. Testing of Hypotheses

In Testing of Hypotheses we consider Null Hypothesis as Confound is not effect and treatments are homogenous. Using ANOVA modules we apply the following steps to verify the Hypotheses condition.

- Grand total G
- Correction factor = G^2 / N ; N= $2*3^2$ = 18
- Block S^2 = S^2_B= $1/3[B_1^2 + \dots B_6^2]$ – C.F.
  = (6 – 1) = 5 deg(f)
- Total S^2T = N-1 = (18-1) =17 deg(f)
- treatments : S^2t= [$S^2A + S^2B + S^2AB + S^2AB^2$]
  = 8 deg(f)
- Error (SSS_E^2) = S^2T - S^2B - S^2t
  = 17 – 5 -8

  = 4 deg(f)

XIV. Balanced Partial Confounding of F.D.

Where the number of replicates in which the effect of the same order are confounded are equal is known as Balanced partially confounded design.

Example -1:

Assuming the 4 replicates of a 2^3 F.D. in 2 Blocks with the effect AB with Replicates I & II. Similarly BC effect confounded in Replicate III & IV.

Replicates →	I		II	
Blocks	1	2	3	4
Confounded effects	AB		AB	

Replicates →	III		IV	
Blocks	5	6	7	8
Confounded effects	BC		BC	

AB and BC are with same order confounded in equal number of replications hence this design is balance.

Exampl-2 :

Assuming the 3 replicates of a 2^3 F.D. in 2 Blocks with the effect AB,ABC,BC with Replicates I, II, and III respectively.

Replicates→	I		II		III	
Blocks	1	2	3	4	5	6
Confounded effects	AB		ABC		BC	

AB and BC are with the same order confounded in same number of replications that is Single replication. ABC is with second order confounded in single replication. Hence this is a balanced partially confounded design.

XV. CONCLUSION

Testing of Hypotheses declares the effective ness of confounding and also the treatments homogenous effects.

If $F_b > F_{d.f(7,6)}$ at α % loss reject Null Hypothesis(H_0), We can consider that confounding is effective.

If Ft > $F_{d.f(7,6)}$ at α % loss reject Null Hypothesis(H_0), that Treatments are not homogenous.

If $F_B > F_{d.f(9,24)}$ at α % loss reject Null Hypothesis(H0), We can conclude that confounding is effective.

If $F_t > F_{d.f(6,24)}$ at α % loss reject Null Hypothesis(H_0), we conclude that treatments are not homogenous and hence proceed to check for the significance of individual effect.

If $F_B > F_{d.f(5,6)}$ at α % loss reject Null Hypothesis(H_0),corresponding to blocks is rejected that is blocks are not homogenous.

It provides that confounding is effective. Similarly If Ft > $F_{(d.f\ 6,6)}$ at α % loss reject Null Hypothesis(H_0), It concludes that the treatments are not homogenous.

$F_b > F_{d.f(7,31)}$ at α % loss reject Null Hypothesis(H_0), then confounding is effective.

If $F_t > F_{d.f(7,31)}$ at α % loss reject Null Hypothesis(H_0), it confirms that treatments are not homogenous then test for the significance of individual effects.

Finally $F_B > F_{d.f(5,4)}$ at α % loss reject Null Hypothesis(H_0), corresponding to confounding is rejected that is confounding is effective.

ACKNOWLEDGMENT

I am very much thankful to the Management and Research Department of Vignan's Institute of Information Technology(Autonomous) for their encouragement in publication Work. I wish to show my gratitude to Prof. K. Srinivasa Rao, Andhra University for motivational thoughts and Support in Research activities.

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